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Laser range finder using Gaussian beam range equation

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ABSTRACT

A new range equation for laser range finder using Gaussian beam profile is proposed. Unlike conventional range equations, the proposed equation can be applied to any ranging conditions, regardless of a beam size of incident light upon targets. We also show an operational coefficient for estimating the returned power using the proposed Gaussian beam range equation as an empirical correction factor.

1. Introduction

One of the useful optical methods for measuring distance is laser range finder (LRF), which has been in use as early as the introduction of lasers [1–4]. Like radar, LRF is based on time-of-flight method. However, instead of radio waves, LRF employs light waves. Both techniques are based on a transmission of a short pulse of electromagnetic radiation and the reception of back scattered signals from a target. The time between the transmission and the reception of the pulse or time of flight t is measured and the distance d is calculated on the basis of the relationship $d=ct/2$, where c is the velocity of light.

In order to measure the time of flight, the transmitting time and the reception time of light pulses are detected by photodiodes of the laser transmitter and the receiver, respectively. The photodiode detecting the transmitted pulse produces an electric signal (start signal), which is used to activate a time counter, while the echo detected by the receiver is for deactivation. Therefore, the range of distance that can be accurately measured by LRF depends on the power of the echo. It is a fact that in general the detected power of the reflected light pulse (echo) is a nonlinear function of the target distance such that the longer the distance, the smaller the detected power. In LRF, a mathematical expression of this relationship is known as range equation. Since light pulse must be reflected by target, this equation is dependent upon characteristics of light pulse and target.

In this paper, we study effects of beam profile on the detected power of the received light pulse. There is discrepancy between the detected powers of light pulse partially and totally reflected by targets [1–3]. Apparently, two different range equations are used for estimating the returned power for different distances. We propose a new range equation, which takes into account Gaussian profile of laser light such that the proposed range equation can be applied to both distances where reflected beams will be partially or totally reflected.

2. Range equations

Let us first consider a point source light transmitter. Since the light spreads in forward hemisphere, its corresponding power at a unit area on the hemisphere can be written as

$$P_{Hemisphere} = \frac{P_{Transmitter}}{2\pi R^2}, \quad (1)$$

where $P_{Transmitter}$ is the power of the transmitted pulse. R is the radius of the hemisphere and the denominator corresponds to the unit area of the hemisphere. The light power arriving at the target is given by

$$P_{Target} = A_{Target}\frac{P_{Transmitter}}{2\pi R^2}, \quad (2)$$

where A_{Target} is the area of the target and R is the distance between the transmitter and the target. Here, R stands for the hemisphere radius and also for the distance range as well. Henceforth, we will refer the distance between the target and transmitter/detector as "range" or R, and the device for measuring the distance is called "range finder".

When the reflected light is detected by a photodiode, the detected power can be mathematically expressed as

$$P_{Detector} = A_{Detector}\frac{P_{Target}}{2\pi R^2}, \quad (3)$$

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where $A_{Detector}$ is the area of the detector. Substitution of Eq. (3) into Eq. (2) yields

$$P_{Detector} = \frac{A_{Detector}\,A_{Target}}{(2\pi R^2)^2} P_{Transmitter}. \tag{4}$$

It is apparent that the detected power is inversely proportional to R^4 of the transmitted power. It is of course difficult to detect the reflected light, since its power is attenuated by $1/R^4$ when it arrives at the detector. The range finder could not be possibly built until the invention of laser. Accordingly, a range finder using a laser is called a "laser range finder" or LRF.

2.1. Geometrical range equations

The reason for using laser for range finder is that a laser can emit high power light, which is almost collimated with a small beam divergence, typically in the order of milliradian. Therefore, with respect to the range to be measured, beam size of the transmitted laser may be either totally or partially reflected by the target. In this section, the range equations for these two conditions is discussed.

Fig. 1 shows a schematic diagram of the range measurement by LRF when the transmitted beam is totally reflected by the target. In order to measure the time of flight, an objective lens with diameter D is used to collect the reflected light onto a photodetector. If the transmitted beam is totally reflected by the target (e.g., R is small), we have

$$P_{Target} = P_{Transmitter}. \tag{5}$$

By regarding the lens diameter as the detector area and taking Eq. (5) into account, Eq. (3) can be rewritten as

$$P_{Detector} = \frac{D^2}{8R^2} P_{transmitter}. \tag{6}$$

However in many cases, for long range measurement, the transmitted laser beam diverges such that its diameter becomes larger than the target dimension. Consequently, the beam is only partially reflected by the target as shown in Fig. 2. In this case, the target reflects only a portion of the transmitted power.

Assume that the transmitted light has a half angle of beam divergence θ. For small θ, the area covered by the light cone at distance R is $\pi(\theta R)^2$ as shown in Fig. 3. The power reflected by the target can be written as

$$P_{Target} = \frac{A_{Target}}{\pi\theta^2 R^2} P_{Transmitter}, \tag{7}$$

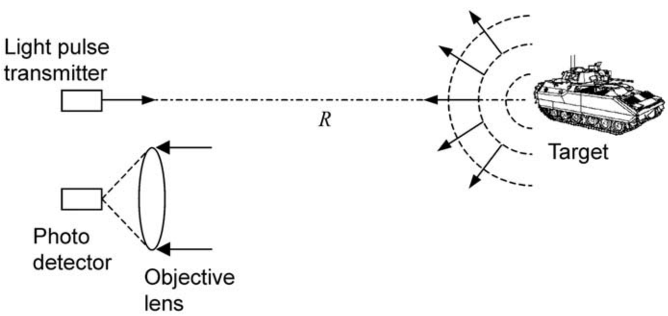

Fig. 1. . Schematic diagram of range measurement using total incidence of pulse laser.

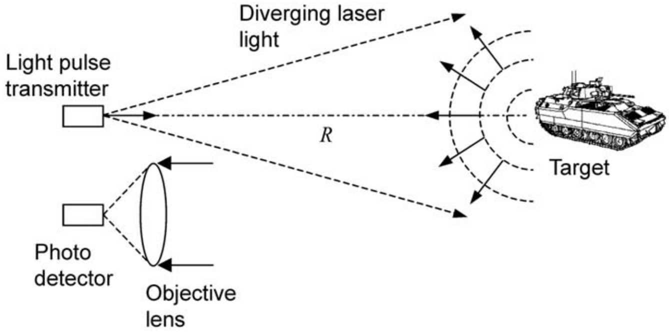

Fig. 2. . Schematic diagram of range measurement using partial incidence of pulse laser.

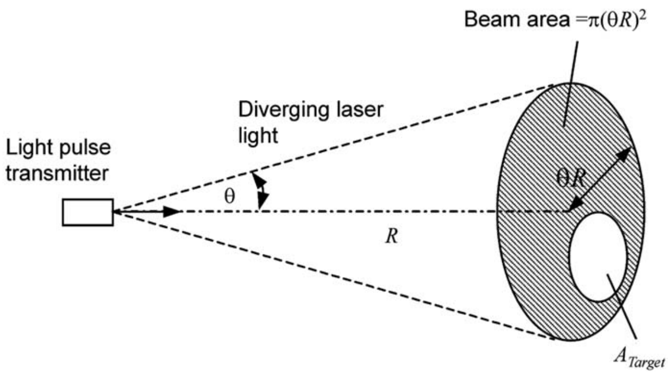

Fig. 3. . Partial incidence of light beam on target.

where A_{Target} is the area of the target that reflects the transmitted light beam. As a consequence, Eq. (6) becomes

$$P_{Detector} = \frac{D^2}{8R^2}\frac{A_{Target}}{\pi\theta^2 R^2} P_{Transmitter} = \frac{D^2 A_{Target}}{8\pi\theta^2 R^4} P_{Transmitter}. \tag{8}$$

It should be noticed that we have assumed that the transmitted power is evenly distributed in a cone. Actually, the light beam is a Gaussian beam, which has peak intensity at the center.

In the above discussion, we have actually assumed that the target is a diffuse object that is subject to Lambert's law [5] and its plane is parallel to the LRF. This implies that the transmitted and the reflected light pulses impinge perpendicularly on the target and the detector, respectively. If the angle between the transmitted beam and the normal of the reflecting target is ϕ, the reflected power must be multiplied with $\cos\phi$. Note that for intensity, it must be $0 \le \cos\phi \le 1$, or $-90^\circ \le \phi \le 90^\circ$. According to the cosine law of Lambert, Eq. (8) must be multiplied with $\cos\phi$, which is the angle between the transmitted beam and the normal of the target

$$P_{Detector} = \frac{D^2 A_{Target}}{8\pi\theta^2 R^4} P_{Transmitter}\cos\varphi. \tag{9}$$

The overall resultant value of $\cos\phi$ may be approximated by 0.5, because the target does not have a flat surface but a topographical structure. Accordingly, Eq. (9) reduces to

$$P_{Detector} = \frac{D^2 A_{Target}}{16\pi\theta^2 R^4} P_{Transmitter}. \tag{10}$$

2.2. Physical range equations

For actual power detection, the geometrical range equation given in Eq. (10) needs to include physical attenuating coefficients such as: target reflectance ρ, optical efficiency of the transmitter $\eta_{Transmitter}$, optical efficiency of the receiver $\eta_{Receiver}$, and atmosphere transmission factor given by

$$T = e^{-\alpha R}, \tag{11}$$

where α is the atmosphere extinction coefficient due to absorption and scattering. Consequently, the physical range equation for a partially reflected beam can be rewritten as

$$P_{Detector} = \frac{T^2 \rho \eta_{Transmitter} \eta_{Receiver} D^2 A_{Target}}{16\pi\theta^2 R^4} P_{Transmitter}. \tag{12}$$

Whereas in accordance with Eq. (6), the physical range equation for the totally reflected beam becomes

$$P_{Detector} = \frac{T^2 \rho \eta_{Transmitter} \eta_{Receiver} D^2}{16 R^2} P_{Transmitter}. \tag{13}$$

Eqs. (12) and (13) are widely known range equations in LRF [1–4].

2.3. Gaussian beam range equation

When a laser beam leaves an optical cavity containing a lasing material, its beam has a Gaussian profile [6]. Let us consider a Gaussian beam propagating in free space along the z direction from z=0. At z=0, the beam waist or beam radius will be at a minimum value w_0. At a large distance z, the beam waist is approximately given by

$$w(z) = \theta z, \tag{14}$$

where θ is the half angle of the beam divergence defined as

$$\theta = \frac{\lambda}{\pi w_0}, \tag{15}$$

where λ is the wavelength of light.

Since the emitted laser beam has a Gaussian shape, the laser beam cannot be an ideally collimated beam. The beam is characterized by its half angle θ or full angle of divergence 2θ. As indicated in Eq. (15), a small beam waist w_0 will produce a large beam divergence θ, and a large beam waist will produce a small beam divergence. Note that the beam waist is the position that the intensity is $1/e^2$ of the center peak intensity.

Assume now that the target we deal with is a circular object with a radius r_{Target}. The target is located at the center of the Gaussian transmitted beam as shown in Fig. 4. The intensity distribution of the Gaussian beam $I(\rho,z)$ is defined as [6]

$$I(\rho,z) = \frac{2P_{Beam}}{\pi w^2(z)} \exp\left[-\frac{2\rho^2}{w^2(z)}\right], \tag{16}$$

where P_{Beam} is the total power carried by the beam given by

$$P_{Beam} = \int_0^\infty I(\rho,z) 2\pi\rho d\rho. \tag{17}$$

The power contained within a circle of radius r_{Target} is

$$P_{Target} = \int_0^{r_{Target}} I(\rho,z)2\pi\rho d\rho = P_{Beam}\left\{1-\exp\left[-\frac{2r_{Target}^2}{w^2(z)}\right]\right\}. \tag{18}$$

By referring to Eq. (14), for large z, such that z=R, the beam waist at this distance can be expressed as below. Thus, Eq. (18) becomes

$$P_{Target} = P_{Beam}\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2 R^2}\right]\right\}. \tag{19}$$

Since P_{Beam} is $P_{Transmitter}$, Eq. (19) can be rewritten as

$$P_{Target} = P_{Transmitterr}\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2 R^2}\right]\right\}. \tag{20}$$

Now we recall Eq. (12) and substitute

$$\frac{A_{Target}}{\pi\theta^2 R^2}$$

with

$$\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2 R^2}\right]\right\}.$$

This yields

$$P_{Detector} = \frac{T^2 \rho \eta_{Transmitter} \eta_{Receiver} D^2}{16 R^2}\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2 R^2}\right]\right\} P_{Transmitter} \tag{21}$$

which is the new Gaussian beam range equation that has never been reported before.

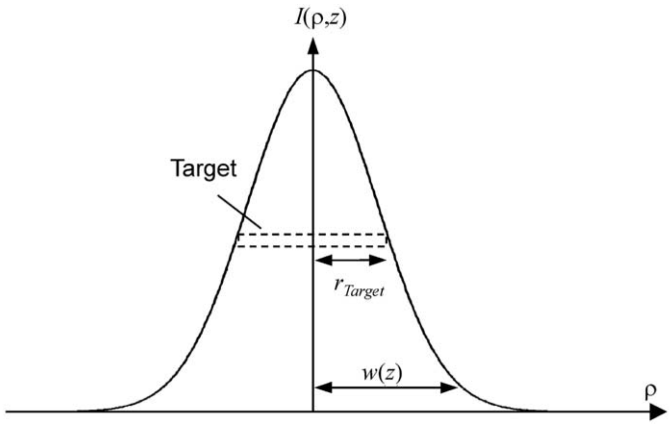

Fig. 4. . Target having a radius r_{Target} at the center of Gaussian beam.

3. Analysis of range equations

We have assembled an eye-safe LRF using MK81 Er:Glass laser from Kigre Inc. (USA) as the transmitter. The emitted laser energy and the pulse width were 3 mJ and 9 ns, respectively. Accordingly, the maximum power of $P_{Transmitter}$ was 0.33 MW. The raw full-angle of divergence of the laser beam was 4.2 mrad. After passing through 4 × Galilean beam expander, the full angle of divergence should be 1.1 mrad. However, from real measurement, we found that the full angle of beam divergence was 1.2 mrad. Thus its half-angle θ was 0.6 mrad. The diameter of the objective lens D was 45 mm. Also from real measurements, we found that the transmitter optical efficiency $\eta_{Transmitter}$ was 0.85, and the receiver optical efficiency $\eta_{Receiver}$ was 0.45.

Some other data were obtained from standard references. The atmospheric absorption coefficient α is 0.1 km^{-1} at clear sky, i.e., visibility 23.5 km. The target reflectance ρ is 0.21 for a green color tank, and the target area is 2.3×2.3 m^2 according to NATO standard. For the Gaussian beam range equation, we assumed that r_{Target}=1.15 m (half of 2.3 m).

We are now in a position to compare three range equations, Eqs. (12), (13) and (21), using numerical data mentioned above. Fig. 5 shows logarithmic plots of the detected powers as a function of the range R. Eq. (21) is represented by a solid curve, while Eqs. (12) and (13) are dot and dash curves, respectively. The

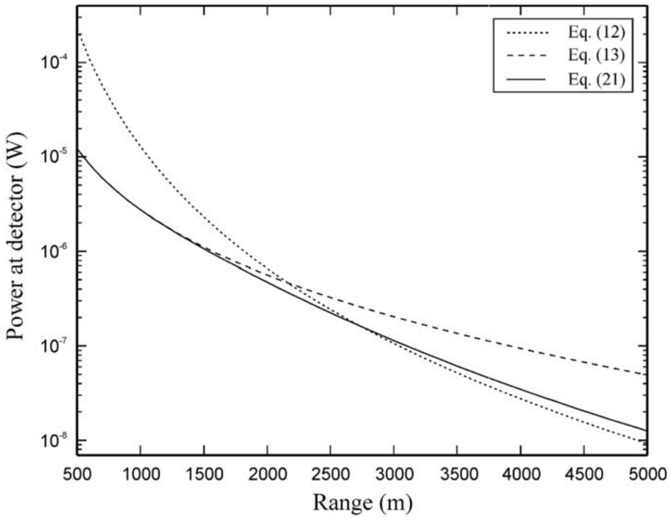

Fig. 5. . Plots of Eqs. (12), (13), and (21).

three plots show the nonlinear relationship between the returned power and the measured range. However, it appears that Eq. (12) is erratic, i.e., when the range is short such that the target area is larger than the light cone, the detected power becomes unrealistically large. Since the target area is 2.3×2.3 m^2 and the half angle divergence is 0.6 mrad, at R=1.9 km, the beam diameter will be about the same as the target width. For R is smaller than 1.9 km, Eq. (12) produces misleading results, while Eqs. (21) and (13) are very close. On the contrary, for R is larger than 1.9 km, Eq. (13) will produce misleading results. It deviates from the plot of Eq. (21), which is approximately the same as that of Eq. (12). Therefore, it is interesting to note that Eq. (21) of the Gaussian beam range equation produces correct results for all values of the measured range R.

In conclusion, Eq. (12), a well known range equation, is valid only for large R or $R > R_0$, and Eq. (13), another well known range equation, which is valid only for small R or $R < R_0$, where R_0 is the range where the transmitted light beam is approximately as large as the target. However, Eq. (21) of the proposed Gaussian beam range equation is valid for all values of R. The advantage of using the Gaussian beam range equation is that we do not need to check its validity by evaluating the target area and the beam diameter as required by conventional range equations of Eqs. (12) and (13). Furthermore, there is no discontinuity when R is approaching R_0 from left ($R > R_0$) and from right ($R > R_0$).

4. Discussion of range equations

Eq. (12) that is proportional to $1/R^4$ is a well-known range equation especially in radar, while Eq. (13) that is proportional to $1/R^2$ is also commonly used for LRF. Both conventional range equations can be found in the reviews by Stitch [1], Bergman [2], and Forrester and Hulme [3]. The use of the range equation proportional to $1/R^4$ for an automotive laser radar system has been reported by Sekine et al. [7]. This equation was also employed for lunar laser ranging with space retroreflector [8].

On the other hand, most LRF papers are related to the equation proportional to $1/R^2$, including a review by Amann et al. [4]. This range equation was not only used in the calculations for LRFs using semiconductor lasers [9–12], but it was also employed in LRFs for military applications [13]. A recent application of this equation to range detections for different atmospheric transmissions was discussed by Steinvall [14].

It is important to note that none of the articles mentioned above presents the range equation based on Gaussian beam modeling similar to the proposed Eq. (21). Apparently, the range equation discussed in each article is related either to $1/R^2$ or $1/R^4$ equation, but not to both. In other words, each subject covers only the range domain of $R < R_0$ or the domain of $R > R_0$, but not both domains.

As shown in Fig. 5, the proposed Gaussian range equation approaches Eq. (13) for $R < R_0$ (R_0=1.9 km), while it approaches Eq. (12) for $R > R_0$. Since the conventional range equations have been verified and widely used in their corresponding domains, our proposed Eq. (21), which unifies Eqs. (12) and (13), is equally and indirectly verified for both domains of $R < R_0$ and $R > R_0$. This is the first reason why experimental verifications of the proposed Gaussian beam range equation were not performed. The second one is due to the fact that the photodetector used for detecting the returned signal has a varying gain (Time Programmed Gain) [15], which is proportional to R^2 for range distance from 20 m to 1 km. Thus, a calculation may be required to get the power prior to the

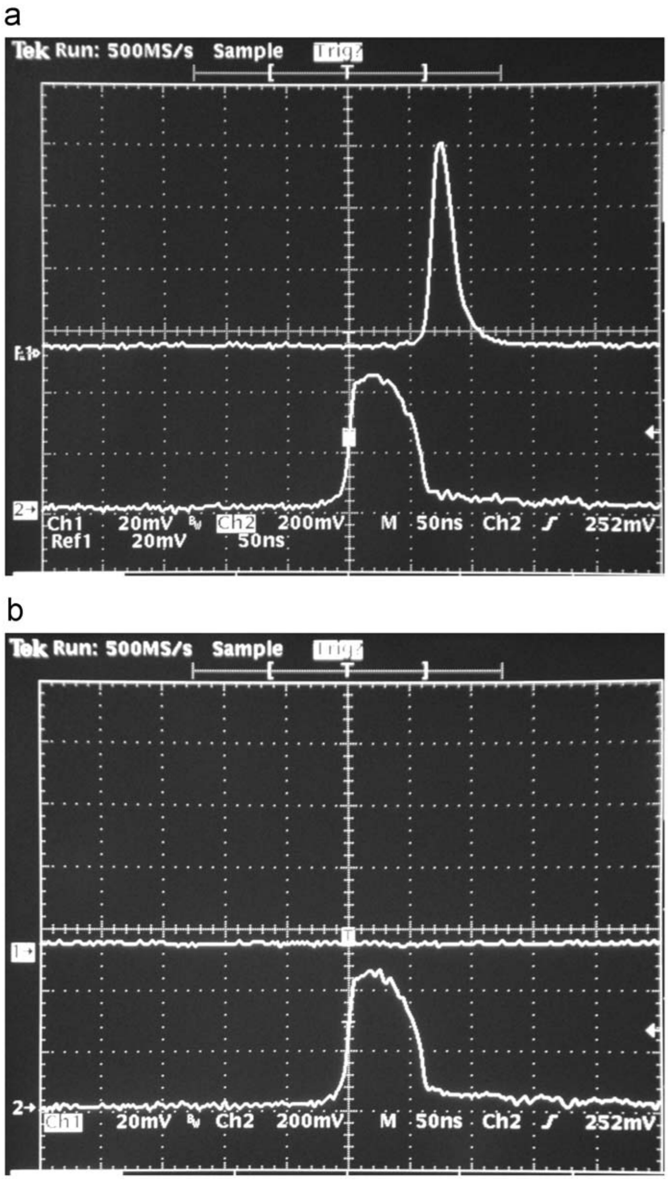

Fig. 6. . Waveforms of the returned signal (upper trace) and the start signal (lower trace) displayed on an oscilloscope for the detected ranges which were (a) smaller and (b) larger than the maximum detectable range, respectively.

Table 1
. Operational coefficient C_{GB} for correction of estimation of the returned power of a laser range finder.

Maximum detectable range R (km)	0.5	1.0	2.0	3.0	4.0
$P_{Detector}$ (W) before correction	1.214×10^{-5}	2.745×10^{-6}	4.726×10^{-7}	1.141×10^{-7}	3.469×10^{-8}
C_{GB}	0.0025	0.01	0.06	0.26	0.87
$P_{Detector}$ (W) after correction	3×10^{-8}	3×10^{-8}	3×10^{-8}	3×10^{-8}	3×10^{-8}

amplifier. At the maximum detectable range, where the signal starts fading away, without referring the gain value, the optical power should be about 30 nW, which is the minimum detectable optical power according to the specification [15].

For illustration of the experimental verifications, Fig. 6 (a) shows waveforms of the returned signal in the upper trace and the start signal in the lower trace of an oscilloscope display. Note that an amplitude of the echo signal has been amplified by the amplifier having varying gain mentioned above. When the detection range is over the maximum of the detectable range, the returned signal vanishes as shown in Fig. 6 (b).

5. Defining operational coefficient

Although we may find accurate D, θ, $\eta_{Transmitter}$ and $\eta_{Receiver}$ from real measurements of LRF, we may not accurately estimate T or α, ρ, and A_{Target} for range equations. In addition, we have approximated the overall resultant of Lambertian coefficient $\cos\phi$ by 0.5. This may not be correct depending on the shape of the target. Thus, to get a correct detected power, we may modify the range equations with a correcting factor, which we may call an operational coefficient or C. Accordingly, Eq. (12) becomes

$$P_{Detector}=C\,\frac{T^2\rho\eta_{Transmitter}\eta_{Receiver}D^2A_{Target}}{16\pi\theta^2R^4}\,P_{Transmitter}, \tag{22}$$

where C is typically between 0 and 1. It is important to note that although the transmitted beam is perfectly aligned with the optical receiver, C may not be 1.

Eq. (22) is applied when the target is smaller than the light beam, or the target reflects partially the transmitted beam. In the case of total reflection of the beam, i.e., the target is larger than the light beam, the detected power is

$$P_{Detector}=C\,\frac{T^2\rho\eta_{Transmitter}\eta_{Receiver}D^2}{16R^2}\,P_{Transmitter}, \tag{23}$$

where the detected power is independent of the target area. Finally, for the Gaussian beam range equation, the detected power becomes

$$P_{Detector}=C\,\frac{T^2\rho\eta_{Transmitter}\eta_{Receiver}D^2}{16R^2}\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2R^2}\right]\right\}P_{Transmitter}. \tag{24}$$

Let us consider that the reflected light pulse is detected by Analog Module Inc. model 759 LRF receiver module [15] having minimum detectable optical power of 30 nW. Therefore, the operational coefficient C can be obtained from

$$C=\frac{30\times10^{-9}}{\frac{T^2\rho\eta_{Transmitter}\eta_{Receiver}D^2}{16R^2}\left\{1-\exp\left[-\frac{2r_{Target}^2}{\theta^2R^2}\right]\right\}P_{Transmitter}}, \tag{25}$$

where the returned optical power detected by this receiver is calculated using the Gaussian beam range equation of Eq. (21) for different maximum detectable target ranges. Table 1 shows the variation of C as a function of maximum detectable range R, which is computed by using data from real measurements and standard references or approximation. The determination of C from real measurement is needed for later use in calibrating or correcting the range equation, which is shown in the last row of the table.

The operational coefficient C is an empirical factor. It is useful for correcting estimation of the detectable range of similar targets, because there are uncertainties in determining atmospheric attenuating factor, reflectivity, size and shape of the target that affects the Lambertian coefficient. It may also include the correction for misalignment between the transmitter optics and the receiver optics. In other applications, a reference target such as a circular plane with known reflectivity and radius can be employed in a test. The resulting operational coefficient C will indicate the quality of the laser range finder.

Although the concept of operational coefficient is well known, this study proposes the use of the new Gaussian beam range equation in determining the operational coefficient C. As shown in Table 1, the maximum detectable ranges may vary from 0.5 to 4 km, the proposed Gaussian beam range equation can be used for R from 0.5 to 4 km. Without using the proposed Gaussian beam range equation, one must investigate whether R is larger or smaller than R_0 (1.9 km) in order to use the correct range equation and its corresponding calibration factor.

6. Conclusions

In this study, we derive, for the first time, a new Gaussian beam range equation, and demonstrate the use of the new Gaussian beam range equation in calculating an operational coefficient for LRFs. Two widely known conventional range equations are exclusively used when (1) the target reflects totally the transmitted beam, and (2) the target reflects partially the transmitted beam. The equation for case (1) will provide erratic results if it is applied to case (2), and vice versa. By using data from our assembled eye-safe LRF, the numerical computations show that the proposed Gaussian beam range equation can be applied to both cases (1) and (2) and provide correct results. Therefore, the proposed equation unifies the two conventional range equations.

References

[1] Stitch ML. Laser rangefinding. In: Arecchi FT, Schulz-Dubois EO, editors. Laser Handbook. Amsterdam: North-Holland; 1972. 1745–1804.
[2] Bergman TG. Design for a Laser Rangefinder. Navweps Report 8674, Nots Tp 3724, Copy 110, U.S. Naval Ordnance Test Station, China Lake, California, December 1964.
[3] Forrester PA, Hulme KF. Laser rangefinder. Opt Quant Elect 1981;13:259–93.
[4] Amann M-C, Bosch T, Lescure M, Myllyla R, Rioux M. Laser ranging: a critical review of usual techniques for distance measurement. Opt Eng 2001;40:10–9.
[5] Moller KD. In: Optics. Mill Valley, California: University Science Books; 1988.
[6] Saleh BEA, Teich MC. In: Fundamentals of Photonics. New York: Wiley; 1991.
[7] Sekine M, Senoo T, Morita I, Endo H. Design method for an automotive laser radar system and future prospect for laser radar. IEEE Int Symp Intelligent Vehicle 1992:120–5.
[8] Yaoheng X, Hesheng F Modification of laser ranging equation. In: Proceedings of the 13th international workshop on laser ranging, Washington, DC, October 7–11, 2002, CD-ROM.
[9] Goldstein BS, Dalrymple GF. Gallium arsenide injection laser radar. Proc IEEE 1967;55:181–8.
[10] Koechner W. Optical ranging system employing a high power injection laser diode. IEEE Trans Aerosp Elect Syst 1968;AES-4:81–91.
[11] Kaisto I, Kostamovaara J, Manninen M, Myllyla R. Optical range finder for 1.5–10 m distance. Appl Opt 1983;22:3258–64.

[12] Burns HN, Christodoulou CG, Boreman GD. System design of a pulsed laser rangefinder. Opt Eng 1991;30:323–9.
[13] Bahuguna KC, Sharma P, Vasan NS, Gaba SP. Laser range sensor. Def Sci J 2007;57:881–90.
[14] Steinvall O. Laser system range calculations and the Lambert W function. Appl Opt 2009;48:B1–7.
[15] Hybrid Eyesafe Laser Rangefinder Receiver. Model 759, Analog Modules, Inc., 126 Baywood Avenue, Longwood, FL, USA.